

Optimal quantum pump in the presence of a superconducting lead

Baigeng Wang¹ and Jian Wang^{1,2,a}

1. Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China

2. Institute of Solid State Physics, Chinese Academy of Sciences, Hefei, Anhui, China

We investigate the parametric pumping of a hybrid structure consisting of a normal quantum dot, a normal lead and a superconducting lead. Using the time dependent scattering matrix theory, we have derived a general expression for the pumped electric current and heat current. We have also derived the relationship among the instantaneous pumped heat current, electric current, and shot noise. This gives a lower bound for the pumped heat current in the hybrid system similar to that of the normal case obtained by Avron et al.

73.23.Ad, 73.40.Gk, 72.10.Bg, 74.50.+r

Since the seminar work of Brouwer,¹ the physics of parametric pumping has attracted increasing attention.²⁻¹⁷ Recently, Avron et al¹⁸ have considered the pumped heat current in the adiabatic regime and found a general lower bound for the heat current. This defines an optimal pump if the heat current equals to the power of Joule heat dissipated during the pumping process¹⁸. As a consequence, the optimal pump is noiseless and charge transported is quantized. The physics of pumped heat current has also been investigated by Moskalets and Buttiker¹⁹ who have derived a general formula for the heat current in the weak pumping regime and the shot noise generated during the pumping process. In the strong pumping regime, the heat current has been studied within the time-dependent scattering matrix theory²⁰ and the existence of optimal pump has been examined. For chaotic quantum dots, Polianski et al²¹ have developed a time-dependent scattering matrix theory to account for the shot noise for parametric pumping and mesoscopic fluctuation for arbitrary temperature and beyond bilinear response. In this paper, we investigate the pumped heat current for a normal superconducting (NS) hybrid system which consists of a normal quantum dot, a normal lead and a superconducting lead. In the adiabatic regime, the energy of charged carriers (electron or hole) is within the superconducting energy gap and hence physics of the Andreev reflection²² dominates. We have derived a general expression for the pumped electric current and heat current in the presence of superconducting lead which is valid at finite pumping amplitude and finite temperature. Our theory is based on the time-dependent scattering matrix theory⁹. Since our theory is perturbative in nature, going beyond the adiabatic regime, one can in principle obtain the pumped electric current and heat current to any order in frequency. In the adiabatic regime, we have also derived a relationship among the instantaneous heat

current, electric current, and the shot noise. This sets a lower bound for the pumped heat current. Similar to the normal system¹⁸, a quantum pump will be optimal if the pumped heat current reaches its lower bound. As a result, the charge transported will be quantized and the system is noiseless just like the normal system. We have also compared with the pumped heat current of NS structure with that of normal structure. For a single pumping potential, the total heat currents generated are the same for NS and normal systems. For two pumping potentials the pumped heat current for NS system can be either larger or smaller than that of normal system depending on the phase difference between two pumping potentials.

For the purpose of presentation, we consider the pumped electric current first. We start with the general definition for the electric current of type α (electron or hole) in the left normal lead in scattering matrix theory ($\hbar = q = 1$),¹⁹

$$I_{e,L\alpha} = \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_0^{\Delta t} dt \langle \hat{I}_{e,L\alpha} \rangle \quad (1)$$

where $\langle \dots \rangle$ denotes the quantum average and $\hat{I}_{e,L\alpha}$ is the electric current operator of type α in the left lead,

$$\hat{I}_{e,L\alpha} = q_\alpha [\hat{b}_{L\alpha}^\dagger(t) \hat{b}_{L\alpha}(t) - \hat{a}_{L\alpha}^\dagger(t) \hat{a}_{L\alpha}(t)] \quad (2)$$

Here the operators $\hat{b}_{L\alpha}$ and $\hat{a}_{L\alpha}$ are annihilation operators for the outgoing and incoming carriers of type α in the left lead and $q_\alpha = 1, -1$ for $\alpha = e, h$. They are related by the scattering matrix,^{19,23}

$$\hat{b}_{L\alpha}(t) = \sum_\beta \int dt' \mathcal{S}_{\alpha\beta}(t, t') \hat{a}_{L\beta}(t') \quad (3)$$

where the time-dependence of the scattering matrix \mathcal{S} is due to the slowly time-varying pumping potential $X(t)$. The distribution function can be obtained by taking the quantum average,¹⁹

$$\langle \hat{a}_{L\alpha}^\dagger(E) \hat{a}_{L\beta}(E') \rangle = \delta_{\alpha\beta} \delta(E - E') f_L(E) \quad (4)$$

where $\hat{a}_{L\alpha}(E)$ is the Fourier transform of $\hat{a}_{L\alpha}(t)$ and $f_L(E)$ is the Fermi distribution function of the left lead. From Eqs.(2), (3), and (4), the pumped electric current is given by

$$I_{e,L\alpha} = \lim_{\Delta t \rightarrow \infty} \frac{q_\alpha}{\Delta t} \int_0^{\Delta t} dt \int dt_1 dt_2 \sum_\beta \mathcal{S}_{\alpha\beta}(t, t_1) f(t_1 - t_2) \mathcal{S}_{\alpha\beta}^*(t, t_2) - q_\alpha \int \frac{dE}{2\pi} f(E) \quad (5)$$

where $f(t) \equiv \int (dE/2\pi) \exp(-iEt) f(E)$. After changing of the variable $t_0 = (t_1 + t_2)/2$ and $\tau = t_1 - t_2$ and using the following Wigner transform for the scattering matrix⁹,

$$\mathcal{S}(t, t') = \int \frac{dE}{2\pi} e^{-iE(t-t')} \mathcal{S}(E, \frac{t+t'}{2}) \quad (6)$$

then Eq.(5) becomes,

$$\begin{aligned} I_{e,L\alpha} &= \lim_{\Delta t \rightarrow \infty} \frac{q_\alpha}{4\pi^2 \Delta t} \int_0^{\Delta t} dt \int dt_0 d\tau dE_1 dE_2 f(\tau) \\ &e^{-iE_1(t-t_0-\tau/2)} e^{iE_2(t-t_0+\tau/2)} \sum_\beta \mathcal{S}_{\alpha\beta}(E_1, \frac{t+t_0}{2} + \frac{\tau}{4}) \\ &\mathcal{S}_{\alpha\beta}^*(E_2, \frac{t+t_0}{2} - \frac{\tau}{4}) - q_\alpha \int \frac{dE}{2\pi} f(E) \end{aligned} \quad (7)$$

Changing the variables again to $\tau_1 = t - t_0$ and $t' = (t + t_0)/2$ and integrating over τ_1 , we obtain

$$\begin{aligned} I_{e,L\alpha} &= \lim_{\Delta t \rightarrow \infty} \frac{q_\alpha}{2\pi \Delta t} \int_{-\Delta t}^{\Delta t} dt' d\tau dE e^{iE\tau} f(\tau) \\ &\sum_\beta \mathcal{S}_{\alpha\beta}(E, t' + \tau/4) \mathcal{S}_{\alpha\beta}^*(E, t' - \tau/4) - q_\alpha \int \frac{dE}{2\pi} f(E) \end{aligned} \quad (8)$$

Using the fact that

$$\begin{aligned} &\lim_{\Delta t \rightarrow \infty} \int_{-\Delta t}^{\Delta t} dt' \sum_\beta \mathcal{S}_{\alpha\beta}(E, t' + \tau/4) \mathcal{S}_{\alpha\beta}^*(E, t' - \tau/4) \\ &= \lim_{\Delta t \rightarrow \infty} \int_{-\Delta t}^{\Delta t} dt \sum_\beta \mathcal{S}_{\alpha\beta}(E, t) e^{-(\tau/2)\partial_t} \mathcal{S}_{\alpha\beta}^*(E, t) \end{aligned} \quad (9)$$

Eq.(8) becomes

$$\begin{aligned} I_{e,L\alpha} &= \frac{q_\alpha}{\pi T_p} \int_0^{T_p} dt \int dE \{ \hat{\mathcal{S}}(E, t) \\ &\times [f(E + i\partial_t/2) - f(E)] \hat{\mathcal{S}}^\dagger(E, t) \}_{\alpha\alpha} \end{aligned} \quad (10)$$

where T_p is the period of the pumping cycle and $\hat{\mathcal{S}}$ is a 2×2 scattering matrix for NS structure with matrix element $\mathcal{S}_{\alpha\beta}$ ($\alpha, \beta = e, h$). Eq.(10) is symbolic and is the central result of this paper. One can in principle obtain the pumped electric current to any order in frequency. For instance, to get the electric current up to ω , it is enough to expand $f(E + i\partial_t/2)$ up to the first order in ∂_t , from which we obtain

$$\begin{aligned} I_{e,L\alpha} &= \frac{iq_\alpha}{2\pi T_p} \int_0^{T_p} dt \int dE \partial_E f \\ &[\partial_t \hat{\mathcal{S}}^\dagger(E, t) \hat{\mathcal{S}}(E, t)]_{\alpha\alpha} \end{aligned} \quad (11)$$

Note that from the unitary condition of the scattering matrix $\hat{\mathcal{S}}$, we have

$$\sum_\beta \mathcal{S}_{\alpha\beta}^* \mathcal{S}_{\alpha\beta} = 1 \quad (12)$$

taking the derivative with respect to time, we obtain

$$\sum_\beta \partial_t \mathcal{S}_{\alpha\beta}^* \mathcal{S}_{\alpha\beta} + c.c. = 0 \quad (13)$$

Hence $\text{Im}[(\partial_t \hat{\mathcal{S}}^\dagger \hat{\mathcal{S}})_{\alpha\alpha}] = -i(\partial_t \hat{\mathcal{S}}^\dagger \hat{\mathcal{S}})_{\alpha\alpha}$. In the adiabatic regime, we have⁹ $\partial_t \mathcal{S}_{\alpha\beta} = \sum_i [\partial_{X_i} \mathcal{S}_{\alpha\beta} \partial_t X_i + \partial_{\dot{X}_i} \mathcal{S}_{\alpha\beta} \partial_t \dot{X}_i + \dots]$ where $\dot{X} \equiv dX/dt$. Up to the order ω , we can neglect the contribution from $\partial_{\dot{X}_i} \mathcal{S}_{\alpha\beta}$. At zero temperature, Eq.(11) becomes,

$$I_{e,L\alpha} = \frac{iq_\alpha}{2\pi T_p} \int_0^{T_p} dt [\partial_{X_i} \hat{\mathcal{S}} \hat{\mathcal{S}}^\dagger]_{\alpha\alpha} \partial_t X_i \quad (14)$$

which agrees with the theory of nonequilibrium Green's function²⁴.

Now we proceed to derive the pumped heat current for NS structure. We note that the heat current is defined as the particle current multiplied by the energy measured from the Fermi level, we thus have from Eq.(10),

$$\begin{aligned} I_{q,L\alpha} &= \frac{1}{\pi T_p} \int_0^{T_p} dt \int dE (E - E_F) \{ \hat{\mathcal{S}}(E, t) \\ &\times [f(E + i\partial_t/2) - f(E)] \hat{\mathcal{S}}^\dagger(E, t) \}_{\alpha\alpha} \end{aligned} \quad (15)$$

Expanding the heat current up to ∂_t^2 and after some algebra, we finally obtained the heat current up to ω^2 ,

$$I_{q,L\alpha} = \frac{-1}{8\pi T_p} \int_0^{T_p} dt \int dE \partial_E f (\partial_t \hat{\mathcal{S}} \partial_t \hat{\mathcal{S}}^\dagger)_{\alpha\alpha} \quad (16)$$

Now we derive the relationship between instantaneous electric current (denoted as $I_e(t) = -iq_\alpha (\partial_t \hat{\mathcal{S}}^\dagger \hat{\mathcal{S}})_{\alpha\alpha}$) and heat current ($I_q(t) = (\partial_t \hat{\mathcal{S}}^\dagger \partial_t \hat{\mathcal{S}})_{\alpha\alpha}$). Now the instantaneous heat current becomes

$$\begin{aligned} I_q(t) &= (\partial_t \hat{\mathcal{S}}^\dagger \partial_t \hat{\mathcal{S}})_{\alpha\alpha} = (\partial_t \hat{\mathcal{S}}^\dagger \hat{\mathcal{S}} \hat{\mathcal{S}}^\dagger \partial_t \hat{\mathcal{S}})_{\alpha\alpha} \\ &= \sum_\beta (\partial_t \hat{\mathcal{S}}^\dagger \hat{\mathcal{S}})_{\alpha\beta} (\hat{\mathcal{S}}^\dagger \partial_t \hat{\mathcal{S}})_{\beta\alpha} \end{aligned} \quad (17)$$

We see that the diagonal term in Eq.(17) is just the $I_e^2(t)$ and the off diagonal term is the shot noise $S_0(t)$ generated during the pumping process^{19,18,25}. Therefore, we have the following relationship

$$I_q(t) = I_e^2(t) + S_0(t) \quad (18)$$

or general lower bound for the heat current

$$I_q(t) \geq I_e^2(t) \quad (19)$$

The condition of optimal pump for NS structure is defined as $S_0(t) = 0$. Following Avron et al¹⁸ it is straightforward to show that the charge transported through the system per cycle is quantized if the quantum pump is optimal.

Now we consider the weak pumping limit for a symmetric double barrier structure in the presence of superconducting lead. The double barrier structure is modeled

by potential $V(x) = X_1(t)\delta(x+a) + X_2(t)\delta(x-a)$ where $X_1(t) = X_0 + X_1 \sin(\omega t)$ and $X_2(t) = X_0 + X_2 \sin(\omega t + \phi)$. In this limit, it is easy to show that the pumped electric current and heat current are given, respectively, by

$$I_{e,L\alpha} = \frac{\omega q_\alpha \sin \phi X_1 X_2}{2\pi} \text{Im}[(\partial_{X_1} \hat{S}^\dagger \partial_{X_2} \hat{S})_{\alpha\alpha}] \quad (20)$$

and

$$I_{q,L\alpha} = \frac{\omega^2}{16\pi} [X_1^2 \partial_{X_1} \hat{S}^\dagger \partial_{X_1} \hat{S} + X_2^2 \partial_{X_2} \hat{S}^\dagger \partial_{X_2} \hat{S} + 2 \cos \phi X_1 X_2 \text{Re}(\partial_{X_1} \hat{S} \partial_{X_2} \hat{S}^\dagger)]_{\alpha\alpha} \quad (21)$$

In Eqs.(20) and (21), we have set $X_i = 0$ in \mathcal{S} after the partial derivatives. Now we will calculate the pumped heat current $I_{q,L\alpha=e}$ for the double barrier NS system. For the NS system, the scattering matrix \mathcal{S}_{ee} and \mathcal{S}_{he} are given by^{22,26}

$$\hat{\mathcal{S}} = \hat{S}_{11} + \hat{S}_{12}(1 - \hat{R}_I \hat{S}_{22})^{-1} \hat{R}_I \hat{S}_{21} \quad (22)$$

where

$$\hat{S}_{ij}(E) = \begin{pmatrix} S_{ij}(E) & 0 \\ 0 & S_{ij}(-E) \end{pmatrix} \quad (23)$$

with S_{ij} being usual scattering matrix for the normal structure. $\hat{R}_I = \alpha \sigma_x$ is the 2×2 scattering matrix at NS interface with off diagonal matrix element α . Here $\alpha = (E - i\nu\sqrt{\Delta^2 - E^2})/\Delta$ with $\nu = 1$ when $E > -\Delta$ and $\nu = -1$ when $E < -\Delta$. In Eq.(22), the energy E is measured relative to the chemical potential μ of the superconducting lead. Eq.(22) has clear physical meaning²⁶. The first term is the direct reflection from the normal scattering structure and the second term can be expanded as $\hat{S}_{12}\hat{R}_I\hat{S}_{21} + \hat{S}_{12}\hat{R}_I\hat{S}_{22}\hat{R}_I\hat{S}_{21} + \dots$ which is clearly the multiple Andreev reflection in the hybrid structure. From Eq.(22) we obtain the well known expressions for the scattering matrix \mathcal{S}_{ee} and \mathcal{S}_{he} ²²

$$\mathcal{S}_{ee}(E) = S_{11}(E) + \alpha^2 S_{12}(E) S_{22}^*(-E) M_e S_{21}(E) \quad (24)$$

and

$$\mathcal{S}_{he}(E) = \alpha S_{12}^*(-E) M_e S_{21}(E) \quad (25)$$

with $M_e = [1 - \alpha^2 S_{22}(E) S_{22}^*(-E)]^{-1}$. In the case of parametric pumping, we assume that the Fermi energy is in line with the chemical potential of superconducting lead, so $E = 0$ and $\alpha = -i$. For the symmetric NS system at resonance, we have $S_{11} = 0$ and $S_{12} = e^{-2ika}$ in the absence of pumping potential. Therefore, from Eqs.(24) and (25), we have

$$\partial_{X_{1/2}} \mathcal{S}_{ee} = \partial_{X_{1/2}} S_{11} - S_{12}^2 \partial_{X_{2/1}} S_{11}^* \quad (26)$$

and

$$\partial_X \mathcal{S}_{he} = -i(\partial_X S_{12}^* S_{12} + c.c.) \quad (27)$$

where we have used the fact that $\partial_{X_1} S_{22} = \partial_{X_2} S_{11}$. Using Fisher-Lee relation²⁷ $S_{\alpha\beta} = -\delta_{\alpha\beta} + iv G_{\alpha\beta}^r$ and the Dyson equation $\partial_{X_j} G_{\alpha\beta}^r = G_{\alpha j}^r G_{j\beta}^r$ ²⁸, we have $\partial_{X_1} S_{11} = iv G_{11}^r G_{11}^r = -i/v$, $\partial_{X_2} S_{11} = iv G_{12}^r G_{21}^r = -i S_{12}^2/v$, and $\partial_{X_{1/2}} S_{12} = -i S_{12}/v$ with the velocity $v = 2k$. Thus from Eqs.(26) and (27), we have $\partial_{X_j} \mathcal{S}_{ee} = 2\partial_{X_j} S_{11}$ and $\partial_X \mathcal{S}_{he} = 0$. From Eq.(21), we obtain

$$I_{q,Le}^{NS} = \frac{\omega^2}{16k^2} [X_1^2 + X_2^2 + 2 \cos \phi X_1 X_2 \cos 4ka] \quad (28)$$

which should be compared with the pumped heat current in the normal case

$$I_{q,L}^N = I_{q,R}^N = \frac{\omega^2}{32k^2} [X_1^2 + X_2^2 + \cos \phi X_1 X_2 (1 + \cos 4ka)] \quad (29)$$

We note that in the NS system, the heat current flows out only through the normal lead while for normal system, the heat current pumps out through both leads. Comparing Eqs.(28) and (29), we have

$$I_{q,L}^N + I_{q,R}^N - I_{q,Le}^{NS} = \frac{\omega^2}{16k^2} \cos \phi X_1 \times X_2 (1 - \cos 4ka) \quad (30)$$

Hence the total pumped heat current generated in the normal system is can be either larger or smaller than that in the NS system depending on the phase difference of two pumping potentials. For a single pump, by setting $X_2 = 0$ in Eqs.(28) and (29), we see that the total pumped heat currents are the same for both NS and normal systems. This is different from the pumped electric current where in the weak pumping regime at resonance, the electric current for NS system is four time larger than that of normal system⁸.

In summary, we have derived a general expression for the pumped electric current and heat current in the presence of superconducting lead using the time-dependent scattering matrix theory. Our theory is valid at finite pumping amplitude and can be applied to the multi-channel systems. Since our theory is perturbative in nature, we can expanding Eq.(15) to the higher order in frequency and hence approach to the nonadiabatic regime. Our theory can also be easily extended to the case of multi-terminal structures. Although our expression is derived for NS system, it is also valid the normal system as well by simply replacing the NS scattering matrix $\mathcal{S}_{\alpha\beta}$ with $\alpha\beta = e, h$ by normal system scattering matrix S_{ij} with $i, j = 1, 2$ in Eqs.(10) and (15). For the NS system, we have found the lower bound for the pumped heat current similar to that of Avron et al¹⁸ for the normal system. As a result, the optimal pump can exist for NS system as well. In the weak pumping limit, we have examined the pumped heat current for NS system for a double barrier structure at resonance. For two parameter pump, we found that the total pumped heat current for

NS structure can be larger or smaller than that of normal structure depending on the phase different between two pumping parameters.

ACKNOWLEDGMENTS

We gratefully acknowledge support by a RGC grant from the SAR Government of Hong Kong under grant number HKU 7091/01P and a CRCG grant from The University of Hong Kong.

^{a)} Electronic mail: jianwang@hkusub.hku.hk

-
- ¹ P.W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
² M. Switkes, C. Marcus, K. Capman, and A.C. Gossard, Science **283**, 1905 (1999).
³ F. Zhou, B. Spivak, and B.L. Altshuler, Phys. Rev. Lett. **82**, 608 (1999).
⁴ M. Wagner, Phys. Rev. Lett. **85**, 174 (2000).
⁵ J.E. Avron, A. Elgart, G.M. Graf, and L. Sadun, Phys. Rev. B **62**, R10618 (2000).
⁶ I.L. Aleiner, B.L. Altshuler, and A. Kamenev, Phys. Rev. B **62**, 10373 (2000).
⁷ Y.D. Wei et al, Phys. Rev. B **62**, 9947 (2000); Phys. Rev. B **64**, 115321 (2001); cond-mat/0207473.
⁸ J. Wang et al, Appl. Phys. Lett. **79**, 3977 (2001).
⁹ M.G. Vavilov et al, Phys. Rev. B **63**, 195313 (2001).
¹⁰ P.W. Brouwer, Phys. Rev. B **63**, 121303 (2001); M.L. Polianski and P.W. Brouwer, Phys. Rev. B **64**, 075304 (2001).
¹¹ X.B. Wang and V.E. Kravtsov, Phys. Rev. B **64**, 033313 (2001).
¹² I.L. Aleiner and A.V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).
¹³ T.A. Shutenko, I.L. Aleiner, and B.L. Altshuler, Phys. Rev. B **61**, 10366 (2000).
¹⁴ Y. Levinson, O. Entin-Wohlman, and P. Wolfle, Physica A **302**, 335 (2001).
¹⁵ M. Moskalets and M. Buttiker, Phys. Rev. B **64**, 201305 (2001).
¹⁶ B.G. Wang, J. Wang, and H. Guo, Phys. Rev. B **65**, 073306 (2002).
¹⁷ J. Wang and B.G. Wang, Phys. Rev. B **65**, 153311 (2002); B.G. Wang and J. Wang, Phys. Rev. B **65**, 233315 (2002); J.L. Wu et al, cond-mat/0204570.
¹⁸ J.E. Avron, A. Elgart, G.M. Graf, and L. Sadun, Phys. Rev. Lett. **87**, 236601 (2001).
¹⁹ M. Moskalets and M. Buttiker, cond-mat/0201259.
²⁰ B.G. Wang and J. Wang, cond-mat/0204067.
²¹ M.L. Polianski, M.G. Vavilov, and P.W. Brouwer, Phys. Rev. B **65**, 245314 (2002).
²² C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).

- ²³ M.P. Anantram and S. Datta, Phys. Rev. B **53**, 16390 (1996).
²⁴ There is a typo mistake in Eq.(2) of Ref. 8 that the minus sign should be the plus sign.
²⁵ When $\hbar\omega \ll k_B T$, the shot noise is $S(t) = S_0(t)/(4\pi k_B T) = |\partial_t S_{he}^* S_{he}|^2/(4\pi k_B T)$.
²⁶ G.B. Lesovik, A.L. Fauchere, and G. Blatter, Phys. Rev. B **55**, 3146 (1997).
²⁷ D. S. Fisher and P. A. Lee, Phys. Rev. B **23**, 6851 (1981).
²⁸ V. Gasparian, T. Christen, and M. Buttiker, Phys. Rev. A **54**, 4022 (1996).